

A SIMPLE AND ACCURATE MODEL FOR PREDICTING MISMATCH EFFECTS IN PHOTOVOLTAIC ARRAYS

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ABSTRACT

In this paper, an accurate and computationally light technique for analyzing the performance of Photovoltaic (PV) arrays is introduced. The proposed approach can be used to quickly assess the achievable Maximum Power Point (MPP) of a PV array. A method for optimally allocating a set of available modules, used to create PV arrays with a given length, is proposed, that allows to maximize the achievable power output of the obtained PV arrays.

Index Terms— Photovoltaic module; Photovoltaic array; Maximum Power Point; Mismatch

1. INTRODUCTION

Photovoltaic Arrays (PV) are nowadays an important technology, witnessing an increasing diffusion due to its role in renewable power generation. Optimizing the power output of a PV array is a very important task, especially in the novel Smart Grid context, that will rely on real time control and redirection of locally available power to loads [1]. The performance modeling of PV arrays has undergone a great deal of research, dating back to the nineties [2][4]. The typical approach to performance modeling problem starts from the individual cell, described by an equivalent circuit involving a current generator, modeling photovoltaic current, one or two diodes, modeling the junction, and series and parallel resistors, modeling interconnection and internal losses respectively. PV modules are described with the same equivalent circuit, but with different parameters [5][6]. Various papers are available in the literature, describing PV cells and modules with both one diode or two diodes equivalent circuits, and proposing approximated models for the corresponding I-V characteristic, obtained by connecting several modules [4]-[12].

Practical PV modules however present one or more bypass diodes, connected in antiparallel with the diodes modeling the semiconductor junction. The function of these diodes is to mitigate negative effects for series connected PV arrays, in presence of partial shading, affecting only part of the modules. Without the bypass diodes, a shaded module, acting as a load for the rest of the PV array, would both upper bound the array current with its own lowered PV current and become a so called hot spot, because the excess current coming from the other arrays would generate heat due to Joule effect. Also this feature has been considered in the literature [2][13].

Most of the proposed models achieve an acceptable accuracy, and are occasionally applied into Maximum Power Point Tracking (MPPT) algorithms [14]. Such techniques however, even those using the Lambert w-function to obtain a more elegant formal description, typically linearize the transcendent implicit equations describing the cell/module equivalent circuit, using numerical techniques such as the Newton-Raphson method [15]-[16]. Hence, the convergence time strongly depends on the initial approximations, and may become exceedingly large when analyzing large PV arrays, especially in presence of heterogeneous modules, with varying equivalent PV currents and diode characteristics.

In this paper, a novel behavioral modeling approach is presented, that, using a deeper behavioral circuit analysis, introduces a very good initial approximation, that in turns guarantees very fast convergence of the following numerical fitting. The proposed approach, validated throughout comparison with circuit simulator results, is suitable for describing large arrays, keeping into account mismatch effects, and may hence provide useful guidelines for optimally allocating a given set of available PV modules, maximizing the achievable Maximum Power Point (MPP).

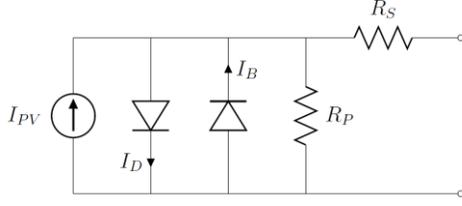


Fig. 1: single diode module equivalent circuit, including bypass diode.

2. SINGLE CELL MODEL AND SERIES ARRAY ANALYSIS

The adopted single cell model is the single diode one, including the bypass diode, as shown in Fig. 1, where I_{PV} is the photovoltaic current. Such model may also be adopted to describe the characteristic of a more complex PV module [5][6]. Throughout this paper, under the hypothesis of an absolute temperature $T=300K$, the following approximated diode model is assumed

$$I \cong \begin{cases} 0, & V < 0.7n \\ I_0 \left(e^{\frac{V}{nV_T}} - 1 \right), & V \geq 0.7n \end{cases} \quad (1)$$

where n is the ideality factor, $V_T=kT/q$, k is the Boltzmann constant, T is the thermodynamic temperature, and q is the electron charge. For different temperatures, the constant 0.7 may vary accordingly.

Consistently, a PV array may then be described by the equivalent circuit in Fig. 2, where the load resistance R_L is shown. In the following, the shunt resistors R_{pi} , $i=1, \dots, N$, are approximated with open circuits, for easiness of analysis. The removal of such hypothesis will be discussed in the final paper. Let us assume $n=n_D$ for the forward diodes, and $n=n_B$ for the bypass diodes, with $n_D \gg n_B$. Let us also assume, without loss of generality, that $I_{PV1} \geq I_{PV2} \geq \dots \geq I_{PVN}$. The threshold based model (1) may be used to derive an approximate description of the array I-V characteristic curve. In particular, it is shown in the following that the I-V characteristic curve is made of a series of constant current regions, linked by steep transients, the current being the PV current I_{PV} of an individual module, and that the breakpoints limiting each plateau correspond to state commutations of the module's diodes. When $V=0$, corresponding to short circuiting the module array with $R_L=0$, the load current is maximum, being equal to I_{PV1} . Moreover, bypass diode B_1 is off, while the remaining bypass diodes B_i , $i=2, \dots, N$, are active, each of them shunting the excess current $I_{PV1}-I_{PV_i}$. The voltage V_1 across module 1 may be evaluated using (1) and the Kirchhoff Voltage Law, obtaining

$$V_1 = V + \sum_{n=2}^N n_B V_T \ln \left(\frac{I_{PV1} - I_{PV_n}}{I_0} + 1 \right) + I_{PV1} R_{ST}, \quad (2)$$

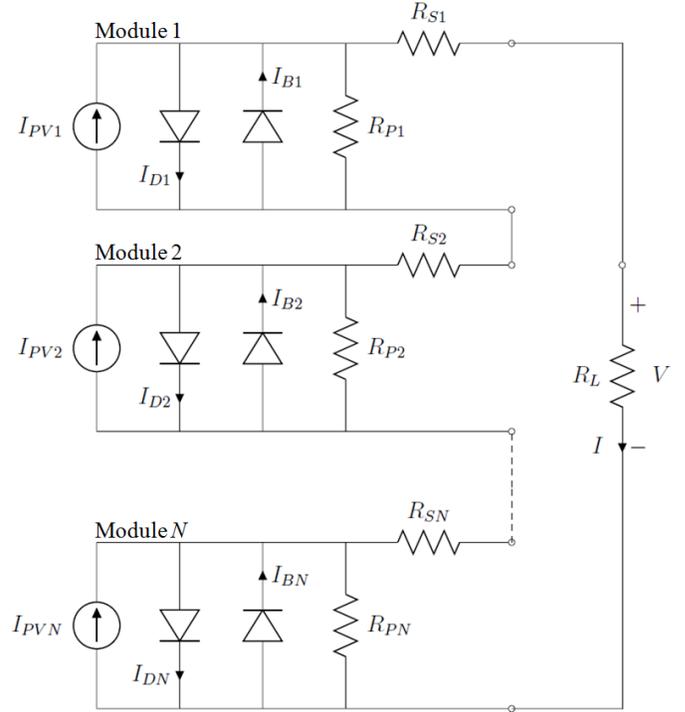


Fig. 2: equivalent circuit of a PV module array.

where $R_{ST} = \sum_{n=1}^N R_{S_n}$. By increasing R_L , the voltage V and

the voltage drop on the bypass diodes in module 1 grow with R_L , as well as V_1 . When V_1 equals $0.7n_D$ the forward diode D_1 switches on, the corresponding load voltage $V_{on,D1}$ being given by

$$V = V_{on,D1} = - \sum_{n=2}^N n_B V_T \log \left(\frac{I_{PV1} - I_{PV_n}}{I_0} + 1 \right) + 0.7n_D - I_{PV1} R_{ST} \quad (3)$$

Notice that, depending on R_{ST} , (3) may provide $V_{on,D1} < 0$. In that case, the short circuit current cannot assume the value I_{PV1} , and should be replaced with the highest I_{PV_m} such that $V_{on,Dm} > 0$.

If R_L is further increased, I is reduced, D_1 shunts a current $I_{PV1}-I$, and the current $I_{PV2}-I$ flowing through B_2 , reduces as well, until B_2 switches off. Under such condition, corresponding to a voltage

$$V = V_{off,B1} = n_D V_T \ln \left(\frac{I_{PV1} - I_{PV2}}{I_0} + 1 \right) + 0.7n_B - \sum_{n=3}^N n_B V_T \ln \left(\frac{I_{PV1} - I_{PV_n}}{I_0} + 1 \right) - I_{PV1} R_{ST}, \quad (4)$$

the circuit current settles to I_{PV2} , and remains approximately constant until a next break point voltage is reached. By further increasing R_L (and V), diode B_3 becomes interdicted, and the circuit current settles to I_{PV3} , remaining constant until D_2 is activated.

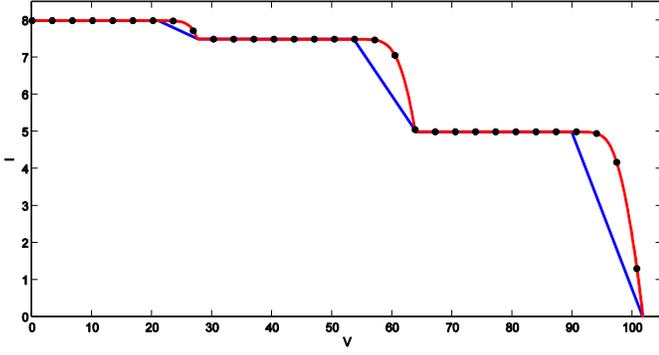


Fig. 3: I - V characteristic of a 3-element PV array. The red curve has been obtained using circuit level simulation, the blue lines describe the initial approximation (5)(6), and the point the following numerical fitting results.

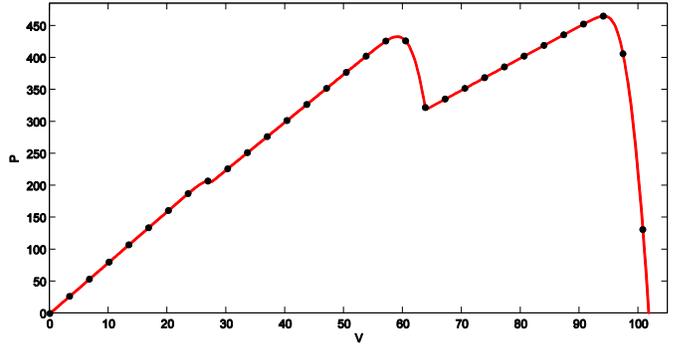


Fig. 4: P - V characteristic of a 3-element PV array. The red curve has been obtained using circuit level simulation, while the point have been obtained using numerical fitting, the initial conditions being set by (5) and (6).

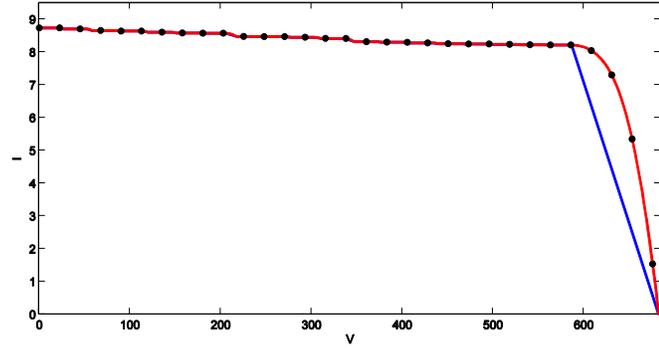


Fig. 5: I - V characteristic of a 20-element PV array. The red curve has been obtained using circuit level simulation, the blue lines describe the initial approximation (5)(6), and the point the following numerical fitting results.

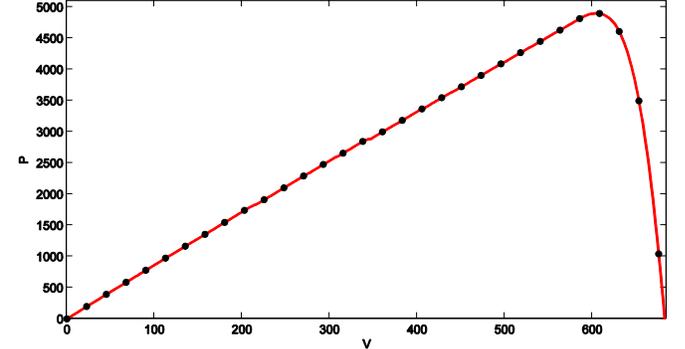


Fig. 6: P - V characteristic of a 20-element PV array. The red curve has been obtained using circuit level simulation, while the point have been obtained using numerical fitting, the initial conditions being set by (5) and (6).

The analysis can be extended iteratively, identifying the breaking points of a piecewise linear approximation of the overall I - V characteristic curve, according to the following circuital behavior. In fact, for growing values of R_L , the current progressively assumes one of the available values of I_{PV} , from the highest (I_{PV1}) to the lowest, in decreasing order. In particular, for each m -th module a *plateau* is defined in the I - V curve, with constant current I_{PVm} in the voltage interval $A_m = [V_{off,Bm}, V_{on,Dm}]$. The left value of such interval, given by

$$V_{off,Bm} = \sum_{n=1}^{m-1} n_D V_T \ln \left(\frac{I_{PVn} - I_{PVm}}{I_0} + 1 \right) - 0.7n_B - \sum_{n=m+1}^N n_B V_T \ln \left(\frac{I_{PVm} - I_{PVn}}{I_0} + 1 \right) - I_{PVm} R_{ST}, \quad (5)$$

$$m = 2, \dots, N$$

corresponds to the deactivation of diode B_m , while the right value, given by

$$V_{on,Dm} = V_{off,Bm} + 0.7(n_D + n_B), \quad m = 1, \dots, N \quad (6)$$

corresponds to the activation of the forward diode D_m . Notice that it follows from (5) and (6), that the amplitude of A_m is $0.7(n_D + n_B)$, for $m=2, \dots, N$. This method may be iterated until the open circuit condition is reached ($V=V_{OC}$, $I=0$). In fact,

when R_L becomes large, each forward diode shunts the photovoltaic current of its own module, obtaining

$$V_{OC} = \sum_{n=1}^N n_D V_T \ln \left(\frac{I_{PVn}}{I_0} + 1 \right). \quad (7)$$

Moreover, the transient linking the last current *plateau* to the ($V=V_{OC}$, $I=0$) can easily be derived from (5)-(6), observing that in such a region all the bypass diodes B are switched off, and all the forward diodes D are switched on, obtaining

$$V = n_D V_T \sum_{n=1}^N \ln \left(\frac{I_{PVn} - I}{I_0} + 1 \right) + I R_{ST}, \quad (8)$$

$$V_{on,D_n} < V \leq V_{OC}$$

The proposed approximations has been used as initial condition to apply Newton-Raphson like fitting to the circuit model, obtaining a very fast and accurate convergence under various conditions. As a first test, a PV array of 3 modules has been analyzed, with $I_0=10^{-15}$ A, $R_{Sn}=0.1\Omega$, $n=1,2,3$, $I_{PV1}=8$ A, $I_{PV2}=7.5$ A, and $I_{PV3}=5$ A respectively. In Fig. 3, the blue line shows the initial I - V curve approximation provided by (5) and (6), the points describe the results of the numerical fitting, while the red line shows the results of a circuit level simulation of the considered PV array, obtained using the QUCS simulator [17]. The adopted selection of the fitting starting point results in a very good agreement

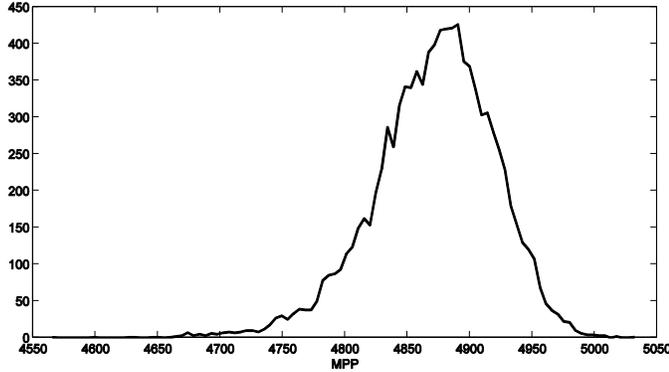


Fig. 7: MPP distribution, evaluated over for a set of 10000 PV arrays, each of them comprising 20 modules. The I_{PV} current of each module is taken from a Gaussian distribution, with a tolerance of 1% about a central value of 8.5A.

between fitting results and circuit simulation, especially near the transition between different current plateaus. This is very important, because, as shown in Fig. 4, where again points describe fitting results and the red curve circuit simulations, the current transitions in Fig. 3 translate into local maxima of the power P delivered to the load. Thus, in absence of accurate modeling MPPT algorithms may erroneously converge to a local maximum, failing to track the MPP [16]. Following such test, more complex PV arrays have been considered, this time featuring $N=20$ modules connected in series, each module photovoltaic current I_{PVn} taken from a normal distribution, with mean value I_{PV0} equal to 8.5A and a standard deviation of $0.03I_{PV0}$, approximately corresponding to a 1% tolerance. Figs 5 and 6 show the obtained $I-V$ and $P-V$ characteristics respectively, where again the blue lines describe the initial approximation based on (5) and (6), the points show fitting results, and the red curve the reference circuit level simulation. Once again, the proposed modeling approach achieves a very good agreement with circuit level results. Notice that, for $N=20$, in absence of an accurate initial approximation, the numerical fitting has been observed to be very time consuming, often failing to converge. Thus, the proposed approach may enable to quickly analyze complex topologies, easily assessing the achievable efficiency and easing their optimization. Such issue is discussed in the following section.

3. ARRAY OPTIMIZATION

Under normal operating conditions, the achievable MPP of a given PV-array is limited by the module with the lowest PV current, because of the series connection. This is true even in presence of bypass diodes, that avoid the occurrence of hot spots and large current reductions due to partial shading, and tolerances on the module fabrication process are expected to introduce mismatches in the PV currents, that may still reduce the achievable MPP. This phenomenon

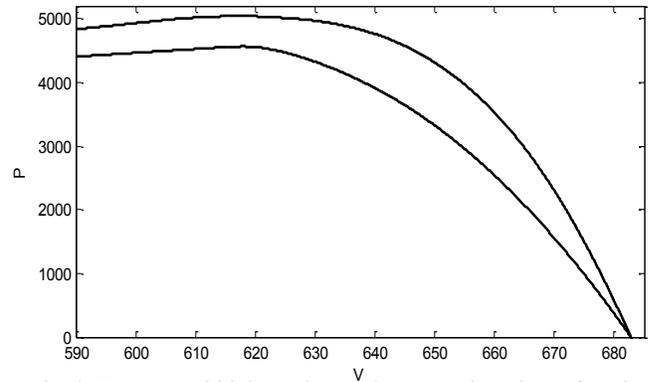


Fig. 8: Lowest and highest observed power, plotted as a function of V , observed in a set of 10000 PV arrays, each of 20 modules. The I_{PV} current of each module is taken from a Gaussian distribution, with a tolerance of 1% about a central value of 8.5A.

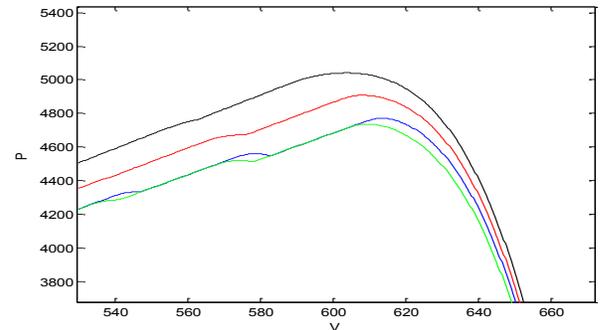


Fig. 9: P-V corresponding to the worst case selection (green and blue for each of the worst case arrays), random selection (red), and average of the P-V curves obtained by applying the proposed selection criterion (black).

has been initially analyzed by means of Montecarlo simulations, with the aim of assessing the effect of PV current variations on the output power. In Fig. 7 a histogram of the observed MPP is reported, obtained for a set of 10000 PV arrays, made of modules that, like that of Fig. 6, realize current the nominal PV current $I_{PV}=8.5A$ with a tolerance of 1%, Gaussian distributed. It can be observed that the also the MPP distribution is approximately Gaussian. Moreover, Fig. 8 plots, as a function of the voltage, the maximum and minimum power observed for each of the considered PV arrays. As expected, the achieved power is sensitive to variations of the PV current, with a perceivable effect on the MPP.

Following such analysis, a simple optimization strategy can be derived, for the problem of optimally allocating a set of M available PV modules in $K=M/N$ subsets of N elements each, subsequently connected in series to obtain M PV arrays. In fact, by assuming that the module with the lowest PV current limits the maximum power, the overall power achievable from the K arrays may be potentially maximized by preliminarily sorting the available modules by available PV current I_{PV} . Then, the PV arrays may be iteratively assembled, taking each time from the sorted set the N best

modules, that is those with the highest I_{PV} , until the last PV array is assembled taking the remaining N worst modules, featuring the lowest I_{PV} . Notice that, according to this criterion, a worst case allocation may be identified as well, that happens when the worst M modules are not grouped, each of them being individually placed in one of the M PV arrays. In order to verify such approach, the problem of allocating $M=40$ modules into 2 arrays of $N=20$ module each has been considered. Fig. 9 shows the P-V curve of an array obtained by randomly selecting 20 modules, in red, the two worst case arrays (blue and green), and the averaged power of the two PV arrays, obtained according to the proposed allocation strategy. It can be observed that, following the proposed approach the output power and the MPP may be significantly increased, with a 6% improvement with respect to the worst case. In the final paper, a model describing the PV array MPP will be presented.

4. CONCLUSIONS

A strategy for quickly and effectively analyzing large PV arrays has been proposed, based on a threshold model that conveniently describes the PV array behavior, effectively selecting the initial conditions, for any subsequent numerical fitting. Validation against circuit level simulations has been carried out. The accuracy of the threshold based analysis guarantees fast convergence of numerical fitting also in presence of large PV arrays, with very complex equivalent circuits. A strategy for maximizing overall output power of PV arrays, minimizing the effects of mismatch between modules, has been proposed and demonstrated. In the final paper, further theoretical results concerning the proposed model will be provided, together with a deeper analysis on the effects of process tolerances on the achievable MPP.

5. ACKNOWLEDGEMENT

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